Gravitational Forces in Dual-Porosity Models of Single Phase Flow

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Abstract—A dual porosity model is derived by the formal theory of homogenization. The model properly incorporates gravity in that it respects the equilibrium states of the medium.

1. Introduction

We consider flow in a naturally fractured reservoir which we idealize as a periodic medium as shown in Fig. 1. There are three distinct scales in this system, the pore scale, the scale of the average distance between fractures, and the scale of the entire reservoir. The concept of dual-porosity [4], [10] is used to average the two finer scales in such a way that the pore scale is recognized as being much smaller than the fracture spacing scale. The fracture system is modeled as a porous structure distinct from the porous structure of the rock (the *matrix*) itself.

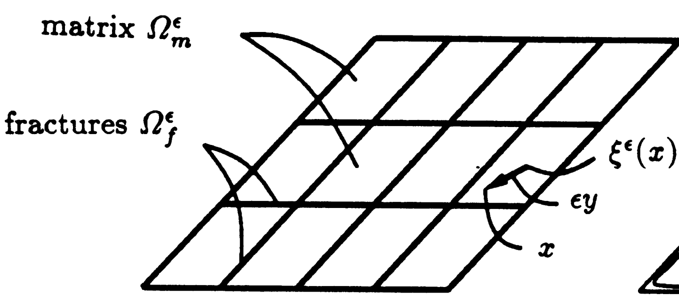

Fig. 1. The reservoir Ω.

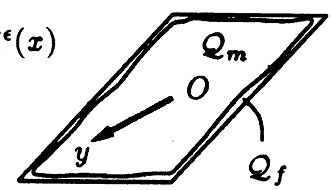

Fig. 2. The unit cell Q.

Dual-porosity models can be derived by the technique of homogenization [2], [3], [6] (see also the general references [5], [7], and [9]). Briefly, we pose the correct microscopic equations of the flow in the reservoir and then let the block size shrink to zero. The resulting macroscopic model is formulated in six space dimensions, three of them represent the entire reservoir over which the fracture system flow occurs. At each point of the reservoir, there exists a three dimensional, "infinitely small" matrix block (surrounded by fractures) in which matrix flow occurs.

For single phase, single component flow, it is recognized that diffusive, gravitational, and viscous forces affect the movement of fluids between the matrix and fracture systems; however, only diffusive forces are easily handled (see, e.g., [1], [4],

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For the matrix,

$$\phi \frac{\partial}{\partial t}\rho(p^\epsilon) - \epsilon\nabla \cdot \left[\mu^{-1}\rho(p^\epsilon)k\big(\epsilon\nabla p^\epsilon - \rho(p^\epsilon)g\mathbf{e}_3\big)\right] = 0, \quad x \in \Omega_m^\epsilon, \tag{4a}$$

$$p^\epsilon = \psi\big(\psi^{-1}(P^\epsilon) + (\epsilon^{-1} - 1)(x_3 - \xi_3^\epsilon(x)) + \bar{\zeta}^\epsilon\big), \quad x \in \partial\Omega_m^\epsilon. \tag{4b}$$

On each $\mathcal{Q}^\epsilon(x)$, we need to define $\bar{\zeta}^\epsilon$. For a given P^ϵ, we can find for each constant $\bar{\zeta}^\epsilon$ the solution $\tilde{p}^\epsilon$ of the steady-state problem corresponding to (4). So, for the given fracture pressure P^ϵ, we take the $\bar{\zeta}^\epsilon$ which gives rise to the $\tilde{p}^\epsilon$ that satisfies

$$\int_{\mathcal{Q}_m^\epsilon(x)} \phi\rho(\tilde{p}^\epsilon)\,dx = \int_{\mathcal{Q}_m^\epsilon(x)} \phi\rho(\bar{p}^\epsilon)\,dx, \tag{5}$$

where $\bar{p}^\epsilon$ is the steady state solution of the unscaled problem corresponding to (4), given by removing the two ϵ's appearing as coefficients in (4a) and replacing (4b) by $\bar{p}^\epsilon = P^\epsilon$. (In the case of an incompressible fluid, simply take $\bar{\zeta}^\epsilon = 0$.)

This ϵ-family of microscopic models satisfies the following:

(i) Darcy flow governs the reservoir, and it does so in the standard way when $\epsilon = 1$ (since then $\bar{\zeta}^\epsilon = 0$);

(ii) For each ϵ, Darcy flow occurs in the fractures and within the *scaled* matrix blocks (i.e., if any matrix block $\mathcal{Q}_m^\epsilon$ is expanded to unit size $\mathcal{Q}_m$, the transformed equations indicate that Darcy flow results);

(iii) If the fracture system is in gravitational equilibrium in the vicinity of a block, then the boundary conditions on that block reflect this gravitational equilibrium;

(iv) For fixed fracture conditions around any matrix block, the steady state matrix solution gives rise to the same mass as calculated from the steady-state solution of the unscaled matrix problem.

We require (iv) so that mass is conserved, since when we scale the matrix problem with (ii)–(iii), we change the pressures which may change the total mass. Under steady-state conditions it is easy to account for any such spurious changes.

We remark that the standard microscopic model [2], [3], [6] replaces (4b) with $p^\epsilon = P^\epsilon$, omits (5), and to be consistent needs to have $\rho(p^\epsilon)g$ replaced by $\epsilon\rho(p^\epsilon)g$ in (3b) and (4a). The novel expression (4b) can be viewed as a scaled continuity of pseudopotential, since we can rewrite it as

$$\psi^{-1}(p^\epsilon) - \big(\xi_3^\epsilon(x) + \epsilon^{-1}(x_3 - \xi_3^\epsilon(x)) + \bar{\zeta}^\epsilon\big) = \psi^{-1}(P^\epsilon) - x_3.$$

The macroscopic model: For the fracture flow,

$$\Phi\frac{\partial}{\partial t}\rho(P^0) + \frac{1}{|\mathcal{Q}|}\int_{\mathcal{Q}_m} \phi\frac{\partial}{\partial t}\rho(p^0)\,dy - \nabla_x \cdot \left[\mu^{-1}\rho(P^0)K\big(\nabla_x P^0 - \rho(P^0)g\mathbf{e}_3\big)\right] = 0, \quad x \in \Omega, \tag{6}$$

for some $\pi(x,t)$, where the $\omega_j(y)$, $j=1,2,3$, are periodic across ∂Q and satisfy

$$-\nabla_y\cdot\left(\nabla_y\omega_j\right)=0,\quad y\in Q_f, \tag{9a}$$

$$\nabla_y\omega_j\cdot\nu=-\mathbf{e}_j\cdot\nu,\quad y\in\partial Q_m. \tag{9b}$$

Recognizing that $(\epsilon^{-1}-1)(x_3-\xi_3^\epsilon(x))\sim(1-\epsilon)y_3$, we have (7) from the ϵ^0 terms of (4).

We now consider (5). First, (4) or (7), without the time derivative term, implies $\tilde{p}^0=\psi\left(\psi^{-1}(P^0)+y_3+\bar{\zeta}^0\right)$. For $\bar{p}^\epsilon$, the ϵ^{-2} terms of its defining equation and the ϵ^0 terms of its boundary condition imply $\bar{p}^0=P^0$. Now a rescaling shows that

$$\int_{Q_m^\epsilon(x)}\phi\rho(\tilde{p}^\epsilon)\,dx\sim\int_{Q_m}\phi\rho\left(\sum_{\ell=0}^{\infty}\epsilon^\ell\tilde{p}^\ell(x,y,t)\right)dy,$$

for some $\tilde{p}^\ell$ depending on the P^ℓ's and on $\bar{\zeta}^\epsilon$. A similar expression holds for the right side of (5), and so the ϵ^0 terms of (5) give the definition of $\bar{\zeta}^0$ as (8).

Finally, the ϵ^0 and ϵ^1 terms of (3a) and (3b) can be analyzed exactly as in the standard model [2], [6] to give (6), and the tensor K is seen to be given by

$$K_{ij}=\frac{K^*}{|Q|}\left(\int_{Q_f}\frac{\partial\omega_j}{\partial y_i}\,dy+|Q_f|\delta_{ij}\right); \tag{10}$$

K is symmetric and positive definite (see, e.g., [3]).

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