Testing Earthquake Damage Prevention Methods with Simulated Fractal Earthquakes

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I. Problem

The goal of this project is to compare several methods of earthquake damage prevention. In order to compare them, I simulated the effects of an earthquake on a building with different earthquake damage prevention methods.

The main problem with earthquakes is that they are very unpredictable. There is no known formula to describe an earthquake. Therefore, we cannot simulate an earthquake by simply programming in a known formula with a regular, predictable behavior.

Irregular, unpredictable phenomena are very common: weather, the economy, etc. To simulate these phenomena Professor Mandelbrot of Yale University invented fractals [6].

II. Simulation of an Earthquake

It is known that an earthquake can be described by fractals [1–6]. It is also known that when we represent a fractal as a sum of monochromatic waves, then the amplitude decreases with frequency ω approximately as: $A \approx \frac{I}{\omega^\alpha}$ for constants I and $\alpha > 0$. So to simulate a random process, we take: $A(\omega_n) = R_n \cdot \frac{I}{\omega_n^\alpha}$, where R_n is a random number simulated by the computer's random number generator, and we also take a random phase. The random phase ϕ_n goes from 0 to 2π (360 degrees in radians).

So the earthquake simulation runs as follows. First, we fix a value $\Delta\omega$, and choose $\omega_n = n\Delta\omega$ (i.e., $\omega_0 = 0, \omega_1 = \Delta\omega, \ldots$). Then we make two random sequences: $R_0, \ldots, R_n$, and $\phi_0, \ldots, \phi_n$. To generate the earthquake force at time t, with N monochromatic waves, we will use the following formula:

$$f_{\text{external}}(t) = \sum_{\text{n}=0}^{N} A(\omega_n)\sin(\omega_n t + \phi_n).$$

III. Simulation of Building Movement

In order to simulate the effect of different controls on the building, we use Newton's second law. According to this law, $f(t) = Ma(t)$, where $f(t)$ is a force, and $a(t)$ is an acceleration. From this formula we can derive the formula for acceleration: $a(t) = \frac{f(t)}{M}$. The force that acts on any object consists of the following components:

- *External forces* that make it move in the first place. It consists of the the actual earthquake force and the control force. We will consider three cases of control force.
  - *No control*, i.e. $f_{\text{control}}(t) = 0$.
  - *Active control* (See, e.g., [7]). The main idea of active control is that if the earthquake pulls the building in one direction active control must pull it in the other direction. The ideal active control is: $f_{\text{control}}(t) = -hx(t)$, but we can't do it like that because the earthquake is very fast and the machine can't react immediately. Therefore there is a delay T: $f_{\text{control}}(t) = -hx(t-T)$.
  - *Hybrid control* (See, e.g., [8–10]). The main idea of hybrid control is this: there is an extra beam in the building that is not attached to anything on one end. When the building moves more than a certain limit a, the beam connects itself to the building. It acts as an extra elastic force. Therefore the formula is as follows: if $x(t-T) < a$ then $f_{\text{control}}(t) = 0$ else $f_{\text{control}}(t) = -hx(t)$ where a is a coefficient [10,11]
- *Elastic force*, this is most visible in a rubber band. This force tries to return the object to its initial position. The farther we bend the object, the stronger is the elastic force. Therefore it is natural to assume that this force is proportional to the displacement: $f_{\text{elastic}}(t) = -kx(t)$, where k is a coefficient.
- *Friction force*, that slows down all of the movement. The faster the movement the bigger the friction force. Therefore it is natural to assume that this force is proportional to the velocity: $f_{\text{friction}}(t) = -nv(t)$, where n is a coefficient.

As a result we get the following formula for force: $f(t) = f_{\text{external}}(t) + f_{\text{elastic}}(t) + f_{\text{friction}}(t) + f_{\text{control}}(t)$.

This research was performed under the supervision of Dr. Nemir at: Department of Computer and Electrical Engineering, University of Texas at El Paso, El Paso, TX 79968

So the algorithm for simulating $x(t)$ is as follows: We start at the moment $t = 0$. We set $x(t) = 0$ and $v(t) = 0$. To get the coordinate and the velocity at the next moment in time we use the following formulas: $x(t + \Delta t) = v(t) \cdot \Delta t + x(t)$, $v(t + \Delta t) = a(t) \cdot \Delta t + v(t)$, where $a(t) = \frac{f(t)}{M}$ and $f(t) = f_{\text{external}}(t) + f_{\text{elastic}}(t) + f_{\text{friction}}(t) + f_{\text{control}}(t)$.

IV. Calculating Energy

Hybrid control does not use any energy. Active control basically eats up lots of energy because it moves the building. Here is how to calculate the energy used.

The bigger the force we apply, the more energy we use. Therefore, it is reasonable to assume that the energy is proportional to force. But if we don't move anything then we don't use any energy. The more we move the building, the more energy we use. So the energy is also proportional to the displacement $\Delta x(t) = x(t + \Delta t) - x(t)$. Therefore $e(t) = f(t) \cdot \Delta x(t)$. To derive how much energy we use altogether we use the following formula:

$$E = \sum_{t=0}^{T} e(t).$$

V. Results and Conclusion

I used the above-described algorithms for earthquake intensities 0.1 to 9.9 on the Richter scale to find out the maximum movement, total movements, and total energy used for no control, active control and hybrid control.

Active control got the smallest total movements on intensities 0.1 through 2.7. Hybrid control got the smallest total movements on the rest of the intensities. So basically hybrid control won because earthquakes with intensities so small do not cause almost any damage.

Active control got the smallest maximum movement on intensities 0.1 through 5.7. Hybrid control got the smallest maximum movement on the rest of the intensities.

And of course hybrid control and no control won the total energy used because they do not use any energy and active control uses a lot.

My conclusion is that hybrid control is the best earthquake damage prevention method.

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